ECON 615: Final Exam

Professor: David Fuller

Monday, December 17, 2012

Instructions:

- You have three hours to complete the exam.
- Read all questions carefully before attempting to answer. If you make any additional assumptions you think are necessary, state clearly what assumptions you are making.
- If you get stuck in the algebra derivations, remember you can get partial marks for explaining the equilibrium concept, the strategy for finding a solution, the results you expect to find, and the intuition for expecting these results.
- Please write **legibly**.

1. This question examines a simple Robinson Crusoe economy. Specifically, Crusoe produces the consumption good using only capital (i.e. labor is inelastically supplied) according to the production function:

$$y_t = f(k_t)$$

Capital depreciates at rate δ. Crusoe has preferences over consumption given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

Each period, from output and un-depreciated capital, Crusoe decides how much to consume, and how much capital to invest for next period (This is exactly the basic Crusoe model we studied in class). Answer the following questions regarding this model.

(a) Write down the Bellman equation determining the solution to this problem. What are the state and control variables?

(b) Find the Euler equation(s) from this recursive formulation and interpret it.

2. Consider the CIA model from class. Recall, we use the Hansen model with indivisible labor, so that preferences are given by:

$$u(c_t^i, h_t^i) = \ln(c_t^i) + Bh_t^i$$

The production function is given by

$$y_t = \lambda_t K_t^\theta H_t^{1-\theta}$$

Here we assume that λ_t is stochastic, and follows the process $\ln(\lambda_{t+1}) = \gamma \ln(\lambda_t) + \varepsilon_{t+1}$. Furthermore, we also assume that the money growth rate remains stochastic, following the process $\ln(g_{t+1}) = (1-\pi)\bar{g} + \pi \ln(g_t) + \varepsilon_t^g$.

Finally, recall that in this model we assume that the agent receives a transfer from the government each period of $(g_t - 1)M_{t-1}$. The agent sells labor to the firm at rate w_t and rents capital at the rate r_t. Given this, the household faces two constraints:

$$\hat{p}_t c_t^i \leq \frac{\hat{m}_{t-1}^i + g_t - 1}{g_t} \tag{1}$$

$$c_t^i + \frac{\hat{m}_t^i}{\hat{p}_t} + k_{t+1}^i = w_t h_t^i + r_t k_t^i + (1-\delta)k_t^i + \frac{\hat{m}_{t-1}^i + g_t - 1}{\hat{p}_t g_t} \tag{2}$$

Answer the following questions regarding this model.

(a) Write the problem down recursively (i.e. write down the Bellman equation). What are the state and control variables?

(b) Using this recursive formulation, show that the household's Euler equations can be written as:

$$\frac{1}{\beta} = E_t\left(\frac{w_t}{w_{t+1}}\left[(1-\delta) + r_{t+1}\right]\right) \tag{3}$$

$$\frac{B}{w_t \hat{p}_t} = -\beta E_t\left(\frac{1}{\hat{p}_{t+1} c_{t+1}^i g_{t+1}}\right) \tag{4}$$

where $\hat{p}_t = \frac{p_t}{M_t}$.

(c) Show that equation (3) can be log-linearized to:

$$0 \approx \tilde{w}_t + \beta E_t \left[\bar{r}(\tilde{r}_{t+1} - \tilde{w}_{t+1}) - (1-\delta)\tilde{w}_{t+1} \right]$$

Now, recall the indivisible labour model of Chapter 6, with no CIA constraint. In that model, the steady state version of (4) is

$$B\bar{c} = -\bar{w}$$

(d) Compare the steady state version of equation (4) with its counterpart in the Chapter 6 version. Explain the effects of the CIA constraint on labour supply, framing your argument in terms of an inflation tax.

3. This question explores a simple OLG model. The population size is fixed; that is, each period one new young agent is born, and an old agent dies. The economy is initially endowed with k_0 units of capital. When young, each agent is endowed with k_t (for a young agent born in period t) units of capital. The agent uses this capital to produce $y_t = f(k_t)$ units of output. This output can be consumed in period t, and the remaining output is used as capital in period $t+1$ to produce $y_{t+1} = f(k_{t+1})$. After producing, the capital k_{t+1} is then endowed to the young born in period $t+1$. To be explicit, each agent has preferences given by:

$$U(c_t^y, c_{t+1}^o) = \ln(c_t^y) + \ln(c_{t+1}^o)$$

and faces the consraints:

$$c_t^y + k_{t+1} = y_t$$

$$c_{t+1}^o = y_{t+1}$$

Further assume that $f(k_t) = k_t^\theta$, $0 < \theta < 1$. Answer the following questions regarding this model.

(a) Find the Euler equation determining the agent's optimal choice of k_{t+1}. Interpret.

(b) Using your Euler equation, find the steady state level of capital.

(c) Now consider the social planner's problem in this economy. Write the planner's problem recursively, find the corresponding Euler equation and steady state level of capital.

(d) Explain intuitively why the allocation in b) is or is not Pareto optimal. If you cannot obtain a solution to the planner's problem, then just explain intuitively how you think is going to be the difference if any.