ECON 615: Final Exam

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Instructions:

- You have three hours to complete the exam.
- The exam is worth a total of 100 points. The weight on each question is given in parenthesis.
- Read all questions carefully before attempting to answer. If you make any additional assumptions you think are necessary, state clearly what assumptions you are making.
- If you get stuck in the algebra derivations, remember you can get partial marks for explaining the equilibrium concept, the strategy for finding a solution, the results you expect to find, and the intuition for expecting these results.
- Please write **legibly**.

1. (30 points) This question explores the Cooley-Hansen CIA model. Recall, we use the Hansen model with indivisible labor, so that preferences are given by:

$$u(c_t^i, h_t^i) = \ln(c_t^i) + Bh_t^i$$

The production function is given by

$$y_t = \lambda_t K_t^\theta H_t^{1-\theta}$$

Here we assume that λ_t is stochastic, and follows the process $\ln(\lambda_{t+1}) = \gamma \ln(\lambda_t) + \varepsilon_{t+1}$. Furthermore, we also assume that the money growth rate remains stochastic, following the process $\ln(g_{t+1}) = (1-\pi)\bar{g} + \pi \ln(g_t) + \varepsilon_t^g$.

Finally, recall that in this model we assume that the agent receives a transfer from the government each period of $(g_t - 1)M_{t-1}$. The agent sells labor to the firm at rate w_t and rents capital at the rate r_t. Answer the following questions regarding this model.

(a) (5 points) Write down the CIA constraint and the household's budget constraint.

(b) (10 points) Show that the household's Euler equations can be written as:

$$\frac{1}{\beta} = E_t \left(\frac{w_t}{w_{t+1}} \left[(1-\delta) + r_{t+1} \right] \right) \tag{1}$$

$$\frac{B}{w_t \hat{p}_t} = -\beta E_t \left(\frac{1}{\hat{p}_{t+1} c^i_{t+1} g_{t+1}} \right) \tag{2}$$

where $\hat{p}_t = \frac{p_t}{M_t}$.

(c) (10 points) Show that equation (2) can be log-linearized to:

$$0 = \tilde{p}_t + \tilde{w}_t - \pi \tilde{g}_t$$

recalling that $\tilde{g}_{t+1} = \pi \tilde{g}_t + \varepsilon^g_{t+1}$

(d) (5 points) Now, the steady state equation for labour is:

$$\bar{H} = \frac{-\beta(1-\theta)(\frac{r}{\theta})}{B(\frac{r}{\theta} - \delta)} \frac{1}{\bar{g}}$$

where recall that $B = \frac{A \ln(1-h_0)}{h_0}$. How does the employment (unemployment) rate change with changes in the average growth rate of money, $\bar{g}$? Explain why, and then draw the implications in a Phillips Curve, with inflation on the horizontal access and employment (or unemployment) on the vertical access. Does this look like a typical Phillips Curve?

2. (30 points) Consider the following OLG model, which is similar to one studied in class. At time $t = 0$, there is a population of N_0 initial old agents, who are each endowed with M_0 units of money. Each period t, N_t young agents are born, where $N_t = nN_{t-1}$. Each agent lives for two periods (i.e. are young and old). When young, agents have an endowment of the consumption good, given by y. When old, they have no endowment. Young and old agents alive in any period t exchange money at price p_t. In addition, the government provides a transfer of g units of consumption to each old agent. This transfer is financed 100% by a

lump-sum tax levied on young agents, τ_t (i.e. the government does NOT issue money here to finance deficits). Agents have preferences given by:

$$U(c_t^y, c_{t+1}^o) = \ln(c_t^y) + \ln(c_{t+1}^o)$$

Answer the following questions regarding this model.

(a) (5 points) Define a competitive equilibrium in this economy. Be sure to specify the government's budget constraint.

(b) (5 points) Assume that $n = 1$, and $N_0 = 1$; i.e. the population size is 1 and there is no population growth. Intuitively argue that p_t is constant.

(c) (10 points) Assuming that $p_t = \bar{p}, \forall t$, find the optimal choice of c_t^y and c_{t+1}^o in equilibrium.

(d) (10 points) Maintaining these assumptions, write down and solve the social planner's problem in this economy. How does it compare with the competitive equilibrium allocations?

3. **Tree-cutting Problem** : (20 points) At time $t = 0$, you plant a tree. Each period, if you do not cut the tree down, it grows. If k_t is the size of the tree in period t, then $k_{t+1} = k_t^\delta$, where $0 < \delta < 1$. You face a simple problem: when to cut the tree down. Assume it is costless to cut the tree down. Your preferences are simple. If you cut the tree down in period t, you enjoy utility k_t; if you do not cut it down in period t, then your utility is simply zero in that period. You discount the future at rate β. Thus, each period, you face the same problem: cut the tree down, or let it grow another period. Answer the following questions about this problem.

(a) (10 points) Write down the Bellman equation determining the solution to this problem.

(b) (10 points) Graphically show the determination of the optimal tree size.

4. (20 points) Consider the Crusoe model with fixed labor supply (from Chapter 4). Specifically, Crusoe solves:

$$\max_{\{c_t, i_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \xi u(c_{t-1})\right]$$

$$\text{s.t.} \quad c_t \leq f(k_t) - i_t, \forall t$$

$$k_{t+1} \leq (1-\delta)k_t + i_t$$

Preferences of this form display *habit persistence*. Answer the following questions regarding this model.

(a) (10 points) Write down the Bellman equation determining the solution to this problem. What are the state and control variables?

(b) (10 points) Find the Euler equations (i.e. F.O.C.) and interpret them.